# EXPERIMENTS FOR TME STUDENT LABORATORY 

GENERAL RADIO COMFANY

No. STX-107

## THE UNIVERSAL IMPEDANCE BRIDGE

1. Why can't we simply measure $L$ and $C$ instead of $L_{\mathbf{s}}$ or $L_{\mathrm{p}}$ and $C_{\mathbf{s}}$ or $C_{\mathrm{p}}$ ?
2. Why does a universal impedance bridge measure $D$ and $Q$ instead of the equivalent series resistance or parallel conductance?
3. When is the null good enough to take advantage of the accuracy built into the bridge?
4. When does the nulling procedure converge rapidly? Is there a cure for an extreme case of "sliding null"?
5. Is there a better balancing procedure than flailing about with the controls?
6. Bridges require a certain amount of skill and knowledge of the operator. Why, then, does one use a bridge instead of one of the direct-reading instruments such as ohmmeters and inductance meters?

## IMPEDANCES AND ADMITTANCES AND THEIR EQUIVALENT CIRCUITS

A two-terminal device can be characterized by either its impedance or its admittance.
$Z=R+j X$
(1) $Y=G+j B$
(2)


$\arg Z=\tan ^{-1} \frac{X}{R}$
(3) $\arg Y=\tan ^{-1} \frac{B}{G}$

Since the device's impedance and admittance are reciprocals of each other, we have
$R+j X=\frac{1}{G+j B}=\frac{G-j B}{G^{2}+B^{2}}$
and
$G+j B=\frac{1}{R+j X}=\frac{R-j X}{R^{2}+X^{2}}$.
(6)

Simply by inspecting these equations and equating real and imaginary parts, we can write down all the relations between the $Z$ - and $Y$ - components:
$R=\frac{G}{G^{2}+B^{2}}$
$G=\frac{R}{R^{2}+X^{2}}$
$X=\frac{-B}{G^{2}+B^{2}}$
(9) $B=\frac{-X}{R^{2}+X^{2}}$.

Notice also that
$\arg Z=\arg \frac{1}{Y}=-\arg Y$.
The characterization of a device in terms of its impedance suggests that we can represent it by an equivalent circuit consisting of a resistance and reactance in series, while the characterization of the same device by its admittance implies an equivalent circuit that consists of a conductance and susceptance in parallel.


Any device can be represented (at a particular frequency) by either the series or parallel equivalent circuit.

Problem: When the rims voltage across the terminals of a particular device is 10 volts, the rms current is 5 amperes, The current leads the voltage by 30 degrees, What are the resistive and reactive components $R$ and $X$ of the device's impedance? What are the conductive and susceptive components $G$ and $B$ of the admittance? Draw the series equivalent circuit, Is the reactance an inductor or a capacitor? Draw the parallel equivalent circuit. Is the susceptance an inductor or a capacitor?

It is important to bear in mind that while any two-terminal physical device can be represented at a given frequency by either the series or parallel equivalent circuit, the equivalent circuit will in general not have the same impedance or admittance as the device at any other frequency. This is because, to take the series equivalent for example, we must expect the impedance of a real device to have a resistive component that varies with frequency, and a reactive component whose magnitude is not proportional to $\omega$ or $1 / \omega$. Obviously we cannot represent such a device at all frequencies by a series resistor and inductor or capacitor whose values do not change with frequency. The most ane can hope for is that one of the equivalent circuits; but not both, may have element values that are fairly constant over part of the frequency range of interest.

When we talk about inductors and capacitors, it is customary to describe the amount of loss in terms not of $R$ or $G$ but of the quality factor $Q$ or dissipation factor $D$. The quantities $Q$ and $D$ convey exactly the same information since they are just reciprocals of each other.
$Q=\frac{1}{D}=\tan |\arg Z|=\frac{|X|}{R}=\tan |\arg Y|=\frac{|B|}{G}$.
The absolute magnitude signs around the angles in (12) mean that $Q$ and $D$ are always positive numbers regardless of whether the device in question is inductive or capacitive. We usually use $Q$ to describe inductors and $D$ to describe capacitors. A low-loss component is one whose impedance (admittance) is almost purely reactive (susceptive) and hence has a high $Q$ and low $D$.

## CAPACITORS

A device is sald to be capacitive when the current through it leads the voltage across it in phase. Thus the impedance of a capacitive device has a negative reactive part and the admittance has a positive susceptive part, and the equivalent circuits are a series resistance $R_{\mathrm{S}}$ and capacitance $\mathrm{C}_{\mathrm{S}}$ or a parallel conductance $G_{p}$ and capacitance $C_{p}$.


$$
\begin{equation*}
Z=R_{\mathrm{S}}-j \frac{1}{\omega \mathrm{C}_{\mathrm{S}}} \tag{13}
\end{equation*}
$$

$$
\begin{equation*}
Y=G_{\mathrm{P}}+j \omega C_{\mathrm{P}} . \tag{14}
\end{equation*}
$$

Referring to the definition (12), we see that the dissipation factor $D$ of a capacitive device is given by
$D=\frac{R_{\mathrm{S}}}{\frac{1}{\omega C_{\mathrm{S}}}}=\frac{G_{\mathrm{P}}}{\omega C_{\mathrm{P}}}$.

Notice that, although $D$ is the same whether we are talking about the series or parallel equivalent circuits, the two formulas for $D$ are different.

If we substitute (15) into (13) and (14) in order to replace $R_{\mathrm{S}}$ and $G_{\mathrm{P}}$ by $D$, we obtain the expressions

$$
\begin{equation*}
Z=\frac{1}{\omega C_{\mathrm{S}}}(D-j) \quad(16) \quad Y=\omega C_{\mathrm{P}}(D+j) \tag{16}
\end{equation*}
$$

Sometimes one wishes to calculate the element values of ane equivalent circuit from those of the other. From (16) and (17) we have
$\frac{1}{\omega C_{\mathrm{S}}}(D-j)=\frac{1}{\omega C_{\mathrm{P}}(D+j)}=\frac{1}{\omega C_{\mathrm{P}}} \cdot \frac{D-j}{D^{2}+1}$.
whence
$C_{S}=C_{P}\left(D^{2}+1\right)$

$$
\begin{equation*}
C_{p}=C_{S} \frac{1}{D^{2}+1} \tag{18}
\end{equation*}
$$

$R_{\mathrm{S}}=\frac{D}{\omega C_{\mathrm{S}}}=\frac{1}{\omega C_{\mathrm{P}}} \cdot \frac{D}{D^{2}+1} \quad G_{\mathrm{P}}=D \omega C_{\mathrm{P}}=\omega C_{\mathrm{S}} \frac{1}{D^{2}+1}$.
(20)
(21)

Question: If $C_{\mathrm{S}}$ and $R_{\mathrm{S}}$ are constant as the frequency varies, will $C_{p}$ and $G_{p}$ also be constant?

Question: Usually we talk about the plain "capacitance" of a capacitor, not $C_{S}$ or $C_{P}$. Strictly speaking, does such a quantity exist? How much difference is there between $C_{S}$ and $C_{P}$ if the capacitor is fairly lossless ( $D \approx 0.001$, say)?

## INDUCTORS

The series and parallel equivalent circuits for inductive devices are

$Z+R_{\mathrm{S}}+j \omega L_{\mathrm{S}}$
(22) $\quad Y=G_{P}-j \frac{1}{\omega L_{P}}$

The Q of an inductive device is given by
$Q=\frac{\omega L_{\mathrm{S}}}{R_{\mathrm{S}}}=\frac{1}{G_{\mathrm{P}} \omega L_{\mathrm{P}}}$
and in terms of $Q$,
$Z=\omega L_{\mathbf{S}}\left(\frac{1}{\mathrm{Q}}+j\right)$

$$
\begin{equation*}
Y=\frac{1}{\omega L_{P}}\left(\frac{1}{Q}-j\right) \tag{25}
\end{equation*}
$$

We can easily obtain the relations between the two equivalent circuits just as we did above in the capacitive case.
$L_{\mathrm{S}}=L_{\mathrm{P}} \frac{1}{\frac{1}{\mathrm{Q}^{2}}+1}$

$$
\begin{equation*}
L_{\mathrm{P}}=L_{\mathrm{S}}\left(\frac{1}{\mathrm{Q}^{2}}+1\right) \tag{27}
\end{equation*}
$$

$R_{\mathrm{S}}=\omega L_{\mathrm{P}} \frac{Q}{\mathrm{Q}^{2}+1}$

$$
\begin{equation*}
G_{P}=\frac{1}{\omega L_{S}} \cdot \frac{Q}{Q^{2}+1} . \tag{29}
\end{equation*}
$$

Question: When we talk about a " $10-\mathrm{mH}$ choke," do we mean that $L_{\mathrm{S}}=10 \mathrm{mH}$ or that $L_{\mathrm{P}}=10 \mathrm{mH}$ ?

Exercise: A particular ferritc bead choke has an impedance at 100 MHz of 115 ohms. If the resistive component of this impedance is 100 ohms, what is $Q$ ? What is $L_{\mathrm{S}}$ ? What are $L_{\mathrm{P}}$ and $R_{\mathrm{P}}$ ?

Question: Ferrite bead chokes look like short circuits at dc. Why is the series ac resistance $R_{\mathrm{S}}$ at 100 MHz so much higher than the dc resistance?

Exercise: Show that

$$
\mathrm{Q}=\frac{1}{2 \omega} \cdot \frac{\text { average energy stored in } L_{\mathrm{S}} \text { or } L_{\mathrm{P}}}{\text { average power dissipated in } R_{\mathrm{S}} \text { or } G_{\mathrm{P}}}
$$

Why does this formula differ by a factor of 2 from the similar formula that gives the $Q$ of a resonant circuit?

## THE BRIDGE BALANCE CONDITION

The general circuit of a bridge is shown in Figure 1. If we assume that the meter impedance is infinite we may regard $Z_{1}$ and $Z_{4}$ as one voltage divider and $Z_{3}$ and $Z_{2}$ as another. It is then apparent that

$$
V_{\text {out }}=\frac{Z_{1}}{Z_{1}+Z_{4}}-\frac{Z_{3}}{Z_{2}+Z_{3}} \cdot V_{\text {gen }}
$$

$$
\begin{equation*}
=\frac{Z_{1} Z_{2}-Z_{3} Z_{4}}{\left(Z_{1}+Z_{4}\right)\left(Z_{2}+Z_{3}\right)} \cdot V_{\text {gen }} . \tag{31}
\end{equation*}
$$



The bridge will "balance" - the output voltage will be zero when the numerator of (31) is zero.
balance condition: $Z_{1} Z_{2}-Z_{3} Z_{4}=0$
Since no current flows in the meter branch at balance, our assumption that the meter impedance is infinite is not necessary for (32) to be valid. The balance condition does not depend on either the generator or the meter.

The complex balance condition (32) yields two real equations. Equating magnitudes and angles, we have
$\left|Z_{1}\right|\left|Z_{2}\right|=\left|Z_{3}\right|\left|Z_{4}\right|$
and
$\arg Z_{1}+\arg Z_{2}=\arg Z_{3}+\arg Z_{4}$.

Equations 33 and 34 must both be satisfied when the bridge is balanced.


FIGURE 2
Equation 34 tells us what combinations of bridge arms have a chance of balancing. For example, the arrangement of Figure 2 can't possibly balance because $\arg Z_{1}$ is positive, $\arg Z_{3}$ is negative, and $\arg Z_{2}$ and $\arg Z_{4}$ are zero, so that $\arg Z_{1}+\arg Z_{2}$ could never equal $\arg Z_{3}+\arg Z_{4}$. The bridge of Figure 3, on the other hand, could balance if the element values were properly adjusted.


## THE SIX BRIDGE CONFIGURATIONS

Accurate variable inductors and capacitors are very costly, so we don't want to use them in a bridge if we can help it. Inductors are never very ideal; they have distributed capacitance, they have low $Q$ at low frequencies, and the resistive part of either of their equivalent circuits is likely to be frequency dependent. Therefore we don't want to use fixed inductors either. With these constraints on the choice of parts to use in the arms, all universal bridges turn out about the same: two variable resistors, a fixed capacitor, and several fixed resistors. In the General Radio Type 1650 Bridge, these
basic components are switched, depending on the nature of the unknown immittance that is to be measured, into one of the configurations shown in Figure 4. Capacitive unknowns whose dissipation factors are not too large are regarded as series-equivalent circuits and the bridge of Figure 4 a measures $C_{\mathrm{Sx}}$ and $D_{\mathrm{x}}$. Capacitive unknowns with large loss are treated as parallel-equivalent circuits and the bridge of Figure 4 b measures $C_{P X}$ and $D_{X}$. Conversely, low-loss inductive unknowns are treated as parallel-equivalent circuits and lossy inductances as series-equivalent circuits. The bridges of Figures 4 c and 4 d measure respectively $L_{\mathrm{Sx}}, \mathrm{Q}_{\mathrm{x}}$ and $L_{\mathrm{Px}}, \mathrm{Q}_{\mathrm{x}}$. Resistance and conductance are measured by the bridges of Figures $4 e$ and $4 f$.


FIGURE 4a



FIGURE 4d


FIGURE 4 e


Let us look at the balance condition of the series capacitance bridge of Figure 4a. Comparison with Figure 1 shows that
$Z_{1}=R_{\mathrm{Sx}}-j \frac{1}{\omega C_{\mathrm{S} x}}$
$Z_{2}=\vec{R}_{\mathrm{CGRL}}$
$Z_{3}=\vec{R}_{D Q}-j \frac{1}{\omega C_{S T}}$
$Z_{4}=R_{R} \quad$,
so that the balance equation 32 becomes
$\left(R_{\mathrm{SX}}-j \frac{1}{\omega C_{\mathrm{SX}}}\right) \vec{R}_{\mathrm{CGRL}}(\mathrm{bal})=\left(\vec{R}_{\mathrm{DQ}}(\mathrm{bal})-j \frac{1}{\omega C_{\mathrm{ST}}}\right) R_{\mathrm{R}}$,
whence, equating real and imaginary parts, we get
$C_{\mathrm{SX}}=\frac{C_{\mathrm{ST}}}{R_{\mathrm{R}}} \vec{R}_{\mathrm{CGRL}} \quad$ (bal)
and
$R_{\mathrm{SX}}=\frac{R_{\mathrm{R}}}{\vec{R}_{\mathrm{CGRL}} \text { (bal) }} \vec{R}_{\mathrm{DQ}}$ (bal).

Equation 37 shows that when the bridge is balanced, $C_{S X}$ is determined from the balance setting of only one of the bridge variables, $\vec{R}_{\mathrm{CGRL}}$; hence the CGRL dial can be calibrated so as to read $C_{\mathrm{SX}}$ directly. But $R_{\mathrm{SX}}$ depends upon the balance settings of both bridge variables, $\vec{R}_{\text {CGRL }}$ and $\vec{R}_{\text {DQ }}$; so we cannot read $R_{\mathrm{SX}}$ directly from a single dial on the bridge. However if we multiply equation 38 by $\omega C_{S \times}$.


FIGURE 5
The General Radio Type 1650-B Impedance Bridge.
and at the same time get rid of $\vec{R}_{\text {CGRL }}$ (bal) by substituting from (37), we have
$R_{\mathrm{S} \times} \omega C_{\mathrm{SX}}=D_{\mathrm{X}}=\omega C_{\mathrm{ST}} \vec{R}_{\mathrm{DQ}}$ (bal)
which shows that at balance the $D$ of the unknown is equal to the $D$ of the $C_{\mathrm{ST}} \vec{R}_{\mathrm{D} Q}$ branch. Since $D_{\mathrm{X}}$ is proportional to the balance setting of $\vec{R}_{D O}$ and also to the frequency, the $D Q$ dial can be provided with a scale that is proportional to $D_{\times}$divided by the frequency. On the Type 1650 Bridge the reading on the series capacitance $D$-scale is equal to $D_{\mathrm{X}} / f(\mathrm{kHz})$.

Equation 39 also helps explain why the series capacitance configuration is not used when $D_{\times}$is large. Large values of $\vec{R}_{\text {DO }}$ would be needed to balance unknowns with large $D^{\prime}$ s. But if $\vec{R}_{\mathrm{DO}}$ is made too large, the unavoidable stray capacitance in the $D Q$ rheostat would introduce a non-negligible shunt capacitance branch into $Z_{3}$, equation 36 would have to be modified, and equation 37 would no longer be valid. Thus $\vec{R}_{\text {DQ }}$ must be limited to reasonable values for the sake of accuracy in the CGRL-dial reading.

Turning now to the parallel-capacitance bridge of Figure 4 b , we see that
$Z_{1}=\frac{1}{G_{P X}+j \omega C_{\mathrm{PX}}}$
$Z_{2}=\vec{R}_{\mathrm{CGRL}}$
$Z_{3}=\frac{1}{\vec{G}_{D Q}+j \omega C_{S T}}$
$Z_{4}=R_{R}$.

The balance equation, which is now

$$
\begin{equation*}
\frac{1}{G_{\mathrm{PX}}+j \omega C_{\mathrm{PX}}} \cdot \vec{R}_{\mathrm{CGRL}}(\text { bal })=\frac{1}{\vec{G}_{\mathrm{DQ}}(\text { bal })+j \omega C_{\mathrm{ST}}} \cdot R_{\mathrm{R}} \tag{41}
\end{equation*}
$$

gives us the two relations
$C_{\mathrm{PX}}=\frac{C_{\mathrm{ST}}}{R_{\mathrm{R}}} \quad \vec{R}_{\mathrm{CGRL}}$ (bal)
and
$G_{\mathrm{PX}}=\frac{\vec{G}_{\mathrm{DQ}} \text { (bal) } \vec{R}_{\mathrm{CGRL}} \text { (bal) }}{R_{\mathrm{R}}}$
or
$D_{\mathrm{X}}=\frac{1}{\vec{R}_{\mathrm{DQ}}(\text { bal }) \omega C_{\mathrm{ST}}}$.

Equation 42 for $C_{P X}$ is identical with equation 37 for $C_{S X}$, so a single scale on the CGRL dial reads both $C_{S \times}$ when the bridge is in the series capacitance configuration and $C_{P X}$ when it is in the parallel capacitance configuration. But equation 44 is not the same as equation 39. This time $D_{\times}$is inversely proportional to the balance setting of $\vec{R}_{D Q}$ and to the frequency. This is why the parallel-capacitance $D$-scale reading on the 1650 is equal to $D \times f(\mathrm{kHz})$, and why its readings increase in the opposite direction from those on the seriescapacitance $D$-scale. Equation 44 also shows that the parallelcapacitance bridge is used for unknowns with large values of $D$ for exactly the same reason that the series capacitance bridge is used for low-D unknowns: to avoid large values of $\vec{R}_{\text {DQ }}$.

Calculations similar to the ones we have just gone through lead to the following balance relations for the seriesand parallel-inductance bridges of Figures $4 c$ and $4 d$, the resistance bridge of Figure 4 e , and the conductance bridge of Figure 4 f.
$\left.\begin{array}{l}L_{\mathrm{SX}}=R_{\mathrm{R}} \mathrm{C}_{\mathrm{ST}} \vec{R}_{\mathrm{CGRL}} \text { (bal) } \\ \mathrm{Q}_{\mathrm{X}}=\omega \mathrm{C}_{\mathrm{ST}} \vec{R}_{\mathrm{DQ}} \text { (bal) }\end{array}\right\} \begin{aligned} & \text { series } \\ & \text { inductance }\end{aligned}$
$\left.\begin{array}{l}L_{\mathrm{PX}}=R_{\mathrm{R}} \mathrm{C}_{\mathrm{ST}} \vec{R}_{\mathrm{CGRL}} \text { (bal) } \\ Q_{\mathrm{X}}=\frac{1}{\omega \mathrm{C}_{\mathrm{ST}} \vec{R}_{\mathrm{DQ}} \text { (bal) }}\end{array}\right\} \begin{aligned} & \text { parallel } \\ & \text { inductance }\end{aligned}$
$R_{\mathrm{X}}=\frac{R_{\mathrm{R}}}{R_{\mathrm{B}}} \vec{R}_{\mathrm{CGRL}}$ (bal) resistance
$G_{\mathrm{X}}=\frac{1}{R_{\mathrm{B}} R_{\mathrm{R}}} \vec{R}_{\mathrm{CGRL}}$ (bal) conductance

## NULLS

Thus far in our discussion we have assumed that the bridge is already balanced; now let us look at the way it gets
balanced. Taking the series-capacitance bridge as our example, we shall calculate the sensitivity of the null to small adjustments of the CGRL and DQ dials.

We suppose that the bridge is very near balance. Let $\vec{R}_{\mathrm{CGRL}}$ and $\vec{R}_{\mathrm{DQ}}$ differ from the balance settings $\vec{R}_{\mathrm{CGRL}}$ (bal) and $\vec{R}_{\mathrm{DQ}}$ (bal) by small amounts $\Delta \vec{R}_{\mathrm{CGRL}}$ and $\triangle \vec{R}_{\mathrm{DQ}}$. If we make the substitution
$\vec{R}_{\mathrm{CGRL}}=\vec{R}_{\mathrm{CGRL}}($ bal $)+\Delta \vec{R}_{\mathrm{CGRL}}$
and
$\vec{R}_{\mathrm{DO}}=\vec{R}_{\mathrm{DO}}(\mathrm{bal})+\triangle \vec{R}_{\mathrm{DO}}$
in the bridge arm impedances (35) and then put these impedances into (31), we shall obtain an expression for $V_{\text {out }}$ as a function of $\triangle \vec{R}_{\text {CGRL }}$ and $\triangle \vec{R}_{\text {DQ }}$.

We can save ourselves considerable work if we exploit the fact that near balance the numerator of equation 31 is almost zero. The differential of a fraction is given by
$d\left(\frac{N}{D}\right)=\frac{D d N-N d D}{D^{2}}$,
and if $N$ is very small we have approximately
$d\left(\frac{N}{D}\right)=\frac{d N}{D} \quad$.
Thus we may ignore the effect of $\Delta \vec{R}_{\mathrm{CGRL}}$ and $\Delta \vec{R}_{\mathrm{DQ}}$ on the denominator of (31). When we substitute (35), (51), and (52) into (31), leaving $\Delta \vec{R}_{\mathrm{CGRL}}$ and $\Delta \vec{R}_{\mathrm{DQ}}$ out of the denominator and noting that most of the terms in the numerator cancel because of the balance condition (36), we have
$V_{\text {out }}=\frac{\left(R_{\mathrm{SX}}-j \frac{1}{\omega C_{\mathrm{SX}}}\right) \Delta \vec{R}_{\mathrm{CGRL}}-R_{\mathrm{R}} \Delta \vec{R}_{\mathrm{DQ}}}{\left(R_{\mathrm{SX}}-j \frac{1}{\omega C_{\mathrm{SX}}}+R_{\mathrm{R}}\right)\left(\vec{R}_{\mathrm{CGRL}}+\vec{R}_{\mathrm{DQ}}-j \frac{1}{\omega C_{\mathrm{ST}}}\right)^{V_{\text {gen }}}}$

The bridge detector does not measure $V_{\text {out }}$ it measures $\left|V_{\text {out }}\right|$. If we multiply equation 53 by its complex conjugate, we can obtain, after considerable arranging of terms and some substitutions from equations 37,38, and 39,

$$
\left|V_{\text {out }}\right|^{2}=\frac{\omega^{2} R_{R}^{2} C_{S X}}{\left[\left(D_{X}+\omega R_{R} C_{S X}\right)^{2}+1\right]^{2}} \times
$$

$$
\begin{equation*}
\left\{\delta_{\mathrm{CGRL}}^{2}+D_{\mathrm{X}}^{2}\left(\delta_{\mathrm{CGRL}}-\delta_{\mathrm{DQ}}\right)^{2}\right\}\left|V_{\mathrm{gen}}\right|^{2} \tag{54}
\end{equation*}
$$

where $\delta_{\mathrm{CGRL}}=\triangle \vec{R}_{\mathrm{CGRL}} / \vec{R}_{\mathrm{CGRL}}$ (bal) and $\delta_{\mathrm{DO}}=\Delta \vec{R}_{\mathrm{DQ}} / \vec{R}_{\mathrm{DO}}$ (bal). Equation 54 expresses the bridge output voltage as a function of the fractional deviations $\delta_{\mathrm{CGRL}}$ and $\delta_{\mathrm{DO}}$ of the $C G R L$ and $D Q$ dials from their balance settings.

Although equation 54 was derived for the series capacitance bridge, essentially the same result is found for the parallel capacitance and series and parallel inductance bridges,

Let us look particularly at the quantity within $\}$ 's in equation 54, for this factor places in evidence the most important characteristics of the null. The occurrence of the product $\delta_{\mathrm{CGRL}} \delta_{\mathrm{DO}}$ shows that the nulling process is not orthogonal, that is to say, a setting of either the CGRL or the $D Q$ dial that gives a minimum bridge output is upset by subsequent adjustment of the other dial. (This phenomenon is called a "sliding null.") Thus, even in principle, a null has to be arrived at through a series of alternate adjustments to the $C G R L$ and $D Q$ dials for minimum output, a process that converges the more slowly the larger the dissipation factor $D_{\mathrm{X}}$. When we adjust $\vec{R}_{\text {CGRL }}$, we find a minimum of $\left|V_{\text {out }}\right|^{2}$ when
$\delta_{\mathrm{CGRL}}=\frac{D_{\mathrm{X}}^{2}}{1+D_{\mathrm{X}}^{2}} \delta_{\mathrm{DO}}$,
and when we adjust $\vec{R}_{\mathrm{DO}}$, this minimum occurs when
$\delta_{D Q}=\delta_{\mathrm{CGRL}}$.

If $D_{\mathrm{X}}$ is very small, the $\delta_{\text {CGRL }}^{2}$ term dominates (54), and the bridge is relatively insensitive to the $D Q$ dial. In this case convergence of the nulling procedure is rapid, since each adjustment of the CGRL dial makes $\mid \delta_{\text {CGRL }} I$ much smaller than $\left|\delta_{\mathrm{DQ}}\right|$, and the succeeding adjustment of the $D Q$ dial makes $\delta_{D O}=\delta_{C G R L}$.

As $D_{\mathrm{x}}$ becomes larger, the influence of the term $D_{\mathrm{x}}^{2}$ $\left(\delta_{\text {CGRL }}-\delta_{\text {DO }}\right)^{2}$ increases and convergence becomes slower. We can see in (55) that each adjustment of the CGRL dial reduces $\mid \delta_{\text {CGRL }}$ l less and less as $D_{\mathrm{X}}$ increases. When $D_{\mathrm{X}}$ is more than about 5 the null can no longer be found at all because the bridge's resolution does not permit us to see a new minimum as we move the CGRL dial away from its previous setting.

The Type 1650 achieves the effect of an orthogonal null for large $D$ (small $Q$ ) values by means of a nonreciprocal mechanical coupling (Orthonul| ${ }^{( }$) between the CGRL and $D Q$ dials. When Orthonull is switched in, a movement of the CGRL dial also moves the DQ dial so as to keep the ratio $\vec{R}_{\text {CGRL }} / \vec{R}_{\text {DO }}$ constant, but a movement of the $D Q$ dial does not affect the CGRL dial. Thus when the CGRL dial is being adjusted, the term $D_{\times}^{2}\left(\delta_{\mathrm{CGRL}}-\delta_{\mathrm{DO}}\right)^{2}$ in equation 54 is constant and the bridge output is sensitive to the term $\delta_{\mathrm{CGRL}}^{2}$, just as though $D_{\mathrm{X}}$ were very small.

## ACCURACY

High loss, high frequency, and either very large or very small unknown impedances decrease the accuracy of the bridge.

One can easily understand why accuracy is adversely affected by loss. Information about the unknown $L$ or $C$ is contained in the imaginary part of the unknown immittance, and when the unknown device is very lossy this imaginary part is swamped by a much larger real part. The decrease in accuracy is thus due to the bridge's limited resolution. When $D_{X}$ is large (say 5 or 10 ), the output voltage is very much more sensitive to the term $D_{X}^{2}\left(\delta_{\mathrm{CGRL}}-\delta_{\mathrm{DO}}\right)^{2}$ than it is to the term $\delta_{\text {CGRL }}^{2}$. Under these circumstances it is impossible to find the true null, where $\delta_{\mathrm{CGRL}}=\delta_{\mathrm{DQ}}=0$, among a neighboring infinity of apparent nulls that are due to the vanishing of $\left(\delta_{\mathrm{CGRL}}-\delta_{\mathrm{DO}}\right)^{2}$.

High frequency decreases accuracy because of stray circuit capacitances and because of inductance in the bridge rheostats. Inspection of the Type 1650 Bridge or its schematic diagram will reveal several devices that have been added to the circuit to compensate for some of these unavoidable reactances.

When the unknown impedance is very large, the bridge's residual capacitance and the capacitance of the leads must be taken into account. Similarly, when the unknown impedance is very small, one must correct for residual bridge and lead inductance.

> Exercise: Add a residual inductance $L_{\mathrm{O}}$ in series with the unknown in the series-capacitance bridge. Write down the new balance equation and derive the formula for the actual dissipation factor of the unknown, $D_{\mathrm{X}}=\omega R_{\mathrm{SX}} C_{\mathrm{SX}}$, in terms of the measured dissipation factor $D_{\mathrm{M}}=\omega \vec{R}_{\mathrm{DQ}} C_{\mathrm{ST}}$.

## SOME EXPERIMENTS

1. Remove the bridge from its case and inspect the parts. Note the 8 adjusting screws that are used to trim the CGRL rheostat's tracking with its dial. Locate the transformer, the standard capacitor, and the metal-film ratio-arm resistors.
2. Balance the bridge with a low-loss capacitor ( $D \leqslant 0.005$ ) connected to the "unknown" terminals. Notice how little the $D Q$ dial affects the balance. How small a deviation from the balance setting of the CGRL dial can you observe? Deliberately introduce an error of about 100 per cent in the $D Q$ dial and readjust the CGRL dial for a minimum meter reading. How much has the CGRL dial reading changed? Can you still detect a 1 per cent error?
3. Connect a low-loss capacitor and a resistor in parallel to make a $D$ of 2 or 3 . Observe the sliding null and the very slow convergence of the nulling procedure without Orthonull. Try again with Orthonull switched in.
4. Measure $L_{\mathrm{S}}$ or $L_{\mathrm{P}}$ and $Q$ of an iron-core inductor, and explain why the measured results depend on the bridge oscillator level.
5. Measure the capacitance of a reverse-biased diode or transistor collector-to-base junction on the $C_{\mathrm{S}}$ bridge as a function of bias voltage, which can be applied through the bridge's bias connection. Note from the bridge schematic that any diode leakage current must flow through the bridge ratio arm. On the $100-\mathrm{pF}$ range, $R_{\mathrm{R}}$ is $1 \mathrm{M} \Omega$, so that $1 \mu \mathrm{~A}$ of leakage current causes a 1 -volt drop in series with the bias supply.
6. Connect a capacitor and resistor in series to obtain a $D$ of about 0.2. Measure $C_{\mathrm{S}}$ and $D$ on the $C_{\mathrm{S}}$ bridge and $C_{\mathrm{P}}$ and $D$ on the $C_{P}$ bridge. Do the two $D$ readings agree? Are the measured $C_{\mathrm{S}}$ and $C_{\mathrm{P}}$ in agreement with (18) and (19)? What should $D$ be at 10 kHz ? Using an external oscillator to drive the bridge, measure $D$ at 10 kHz .


|  | $p F$ | $n F$ |  |  | $\mu \mathrm{~F}$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MULT | 100 | 1 | 10 | 100 | 1 | 10 |  |
| RA | 1 M | 100 k | 10 k | 1 k | 100 | 10 |  |

a. Turn GENERATOR switch to BAT CHECK position. If the meter pointer is not in the BAT sector, replace the batteries.
b. Turn GENERATOR switch to AC EXTERNAL or AC INTERNAL 1 kHz .
c. Turn PARAMETER switch to $\mathrm{C}_{\mathrm{s}}$.
d. Connect the unknown so that most stray capacitance is between the LOW terminal and the $1650-B$ case.
e. Turn ORTHONULL ${ }^{\circledR}$ switch to OUT.
f. Turn OSC LEVEL clockwise. The panel control affects only the internal oscillator.
g. Turn DQ dial near 0.05 on the LOW D scale.
h. Turn CGRL dial near 11 .
i. Adjust DET SENS for about 6 divisions deflection.
j. Turn MULTIPLIER switch for minimum meter reading.
k. Alternately adjust, first the CGRL dial, then the $D Q$ dial for the best null, increasing the DET SENS as needed.

1. ORTHONULL ${ }^{(8)}$ is not used on this bridge unless the DQ dial reading times $f(\mathrm{kHz})$ approaches or exceeds 1.
$m$. If the $D Q$ dial goes into the uncalibrated portion, the unknown should be measured as $C_{p}$.
$n$. The series capacitance of the unknown equals the product of the CGRL-dial reading and the MUL-TIPLIER-switch setting.

- The $D$ equals the reading on the DQ dial times $f\left(\mathrm{kHz}_{\mathrm{z}}\right)$.
p. Turn GENERATOR switch to OFF.

a. Turn GENERATOR switch to BAT CHECK position. If the meter pointer is not in the BAT sector, replace the batteries.
b. Turn GENERATOR switch to A.C EXTERNAL or AC INTERNAL 1 kHz .
c. Turn PARAMETER switch to $\mathrm{C}_{\mathrm{p}}$. Large electrolytics should be measured at a low frequency ( 120 Hz ) for greater accuracy.
d. Connect the unknown so that most stray capacitance is between the LOW terminal and the $1650-\mathrm{B}$ case.
e. Turn ORTHONULL ${ }^{(8)}$ switch to OUT.
f. Turn OSC LEVEL clockwise. The panel control affects only the internal oscillator.
g. Turn DQ dial near 0.2 on the HIGH D scale.
h. Turn CGRL dial near 11 .
i. Adjust DET SENS for about 6 divisions deflection.
j. Turn MULTIPLIER switch for minimum meter reading.
k. Alternately adjust, first the $D Q$ dial, then the CGRL dial for the best null, increasing the DET SENS as needed.
I. ORTHONULL ${ }^{\circledR}$ switch should be set to $\mathbb{N}$ if the $D Q$ dial reading times $1 / \mathrm{f}(\mathrm{kHz})$ approaches or exceeds 1.
m . If the $D Q$ dial reaches the stop at 0.1 , the unknown should be measured as $C_{5}$.
n . The parallel capacitance of the unknown equals the product of the CGRL-dial reading and the MUL-TIPLIER-switch setting.
o. The $D$ equals the reading on the DQ dial times $1 / \mathrm{f}(\mathrm{kHz})$.
p. Turn GENERATOR switch OFF.

R


|  | m |  | $\ldots$ |  | k |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MULT | 100 | 1 | 10 | 100 | 1 | 10 | 100 |
| Raf | 1 | 10 | 100 | 1 k | 10k | 100k | 1 M |

a. Check mechanical zero of meter. b. Turn GENERATOR switch to the BAT CHECK position. If the meter pointer is not in the BAT sector, replace the batteries.
c. Turn GENERATOR switch to the desired generator source. The OSC LEVEL control affects only the internal oscillator.
d. Turn ORTHONULL® switch to OUT and PARAMETER switch to R.
e. Turn CGRL dial near 11.
f. Adjust DET SENS control for about 6 divisions deflection.
g. Turn MULTIPLIER switch for minimum reading to the left of center if making a dc measurement. Null as usual if making an ac measurement. (DQ rheostat not in the circuit.)
h. Adjust CGRL dial for best ac null, or zero the pointer if using dc. If ac null is not sharp, a reactive balance may be necessary, see instruction manual.
i. The unknown resistance is the CGRL-dial reading multiplied by the MULTIPLIER switch setting. i. Turn GENERATOR switch to OFF.

## OPERATING INSTRUCTIONS


a. Check mechanical zero of meter. b. Turn GENERATOR switch to the BAT CHECK position. If the meter pointer is not in the BAT sector, replace the batteries.
c. Turn GENERATOR switch to the desired generator source. The OSC LEVEL control affects only the internal oscillator.
d. Turn ORTHONULL® ${ }^{(8)}$ switch to OUT and PARAMETER switch to $G$.
e. Turn CGRL dial near 11 .
f. Adjust DET SENS control for about 6 divisions deflection.
g. Turn MULTIPLIER switch for minimum reading to the left of center if making a dc measurement. Null as usual if making an ac measurement. (DQrheostat not in the circuit.) h. Adjust CGRL dial for best ac null, or zero the pointer if using dc, If ac null is not sharp, a reactive balance may be necessary, see instruction manual.
i. The unknown conductance is the CGRL-dial reading multiplied by the MULTIPLIER switch setting. i. Turn GENERATOR switch to OFF.

o. Turn GENERATOR switch to BAT CHECK. If the meter pointer isn't in the BAT sector, replace the batteries.
b. Turn GENERATOR switch to AC EXTERNAL or AC INTERNAL 1 kHz . Air core if chokes should be measured at a high frequency ( 10 kHz ) to get a reasonable Q .
c. Turn PARAMETER switch to $L_{s}$. d. Connect unknown so that most stray capacitance is between the LOW terminal and the $1650-\mathrm{B}$ case. e. Turn ORTHONULL ${ }^{(8)}$ switch to OUT.
f. Turn OSC LEVEL clockwise, The panel control affects only the internal oscillator. Use full output except for nonlinear unknowns. Iron core inductors are often nonlinear.
g. Turn DQ dial near 4 on the LOW Q scale.
h. Turn CGRL dial near 11.
i. Adjust DET SENS for about 6 divisions deflection.
j. Turn MULTIPLIER switch for minimum meter reading.
k. Alternately adjust the CGRL and DQ dials for the best null, DQ dial first, increasing the DET SENS as needed. Null means bring the pointer as near to the center of the meter as possible. Usually it won't be possible to center the pointer. I. ORTHONULL ${ }^{\circledR}$ should be switched $\mathbb{N}$ if the $D Q$-dial reading times $\mathrm{f}(\mathrm{kHz})$ approaches or is less than 1 . m. If a sharp null cannot be obtained and the $Q$ dial is near 10 , switch to $L_{p}$.
n . The series inductance of the unknown equals the product of the CGRL-dial reading and the MULTI-PLIER-switch setting.
o. The $Q$ of the unknown equals the Q-diol reading times $f(\mathrm{kHz})$.
p. Turn GENERATOR switch OFF.

a. Turn GENERATOR switch to BAT CHECK. If the meter pointer isn't in the BAT sector, replace the batteries.
b. Turn GENERATOR switch to AC EXTERNAL or AC INTERNAL 1 kHz .
c. Turn PARAMETER switch to $L_{p}$.
d. Connect unknown so that most stray capacitance is between the LOW terminal and the 1650-B case. e. Turn ORTHONULL ${ }^{\circledR}$ switch to OUT.
f. Turn OSC LEVEL clockwise. The panel control affects only the internal oscillator. Use full output except for nonlinear unknowns. Iron core inductors are often nonlinear.
g. Turn DQ dial near 5 on the HIGH $Q$ scale.
h. Turn CGRL dial near 11.
i. Adjust DET SENS for about 6 divisions deflection.
j. Turn MULTIPLIER switch for minimum meter reading.
k. Alternately adjust the CGRL and DQ dials for the best null, CGRL first, increasing the DET SENS as needed. Null means bring the pointer as near to the center of the meter as possible. Usually it won't be possible to center the pointer.

1. ORTHONULL® is not used on this bridge unless the $D Q$ dial reading times $1 / f(\mathrm{kHz})$ approaches 1 or less.
m. If a sharp null cannot be obtained, the unknown is too lossy and must be measured as $L_{s}$, or the unknown is not inductive.
n . The parallel inductance of the unknown equals the product of the CGRL-dial reading and the MULTI-PLIER-switch setting.
o. The Q of the unknown equals the dial reading times $1 / f(\mathrm{kHz})$.
p. Turn GENERATOR switch to OFF.

- measures L, C, and loss; R and G
- $1 \%$ accuracy

The 1650 Impedance Bridge will measure the inductance and storage factor, $Q$, of inductors*, the capacitance and dissipation factor, D, of capacitors, and the ac and dc resistance or conductance of resistors.

In the laboratory it is extremely useful for measuring the circuit constants in experimental equipment, testing


## specifications

- 20 Hz to 20 kHz , internal 1 kHz and dc
- portable, self-contained, battery-operated
preliminary samples, and identifying unlabeled parts. In the shop and on the test bench it has many applications for testing and component sorting.

Three-terminal measurements can be made in the presence of considerable stray capacitance to ground.

## DESCRIPTION

This bridge is completely self-contained and portable. Battery-powered, low-drain transistor oscillator and detector are included. The panel meter indicates both dc and ac bridge unbalances.

The measured quantities, R, G, L, C, D, and Q, are indicated directly on dials with constant-percentage-accuracy logarithmic scales. Multiplier and the units of measurement are indicated by the range setting.

The bridge circuit elements are high-quality, stable components that ensure long-term accuracy. The Orthonull balance finder, a patented mechanical-ganging device, is used to make a low-Q (high-D) balance possible without a sliding null. This mechanism, which may be switched in or out as desired, adds accuracy as well as convenience to low-Q measurements that are practically impossible on other impedance bridges.

The Flip-Tilt case provides a handie and a captive, protective cover that allows the bridge panel to be tilted and held firmly at any angle.

* Including such low-Q inductors as if coils measured at 1 kHz .

RANGES OF MEASUREMENT

|  | 20 Hz to 20 kHz + | DC | Residuals |
| :---: | :---: | :---: | :---: |
| ```Capacitance 1 pF to 1100 \mu\textrm{F}}\mathrm{ , series or parallel, 7 ranges``` | $\pm 1 \% \pm 1 \mathrm{pF}$ |  | $=0.5 \mathrm{pF}$ |
| Inductance <br> $1 \mu \mathrm{H}$ to 1100 H , series or parallel, 7 ranges | $\pm 1 \% \pm 1 \mu \mathrm{H}$ |  | $=0.2 \mu \mathrm{H}$ |
| ```Resistance ac or dc, 1 m\Omega to 1.1 M\Omega, 7ranges``` | $\pm 1 \% \pm 1 \mathrm{~m} \Omega$ | $\pm 1 \%, 1 \Omega$ to $100 \mathrm{k} \Omega$, ext supply or detector required $>100 \mathrm{k} \Omega$ and $<1 \Omega$. | $=1 \mathrm{~m} \Omega$ |
| Conductance ac or de, 1 nanomho to 1.1 mhos, 7 ranges | $\pm 1 \% \pm 1$ nanomho | $\pm 1 \%, 10$ micromhos to 1 mho, ext supply or detector required $<10$ micromhos. |  |
| Dissipation Factor, D, at 1 kHz , 0.001 to 1 of series $C_{\text {, }}$ 0.1 to 50 of parallel C . | $\pm 5 \% \pm 0.001$ at 1 kHz and lower |  |  |
| Storage Factor, Q, at 1 kHz , 0.02 to 10 of series L, 1 to 1000 of parallel L. | $\begin{aligned} & \frac{1}{Q} \text { accurate to } \\ & \frac{ \pm 5 \%}{1} \mathrm{kHz} \pm 0.001 \text { at lower } \end{aligned}$ |  |  |

\& Bridge operates up to 100 kHz with reduced accuracy.

## GENERAL

Generator: Internal; $1 \mathrm{kHz} \pm 2 \%$. Type 1310 or 1311 Oscillator recommended if external generator is required. Internal dc supply, $6 \mathrm{~V}, 60 \mathrm{~mA}$, max.
Detector: Internal or external; internal detector response flat or selective at 1 kHz ; sensitivity control provided. Type 1232-A Tuned Amplifier and Null Detector is recommended if external detector is required. Combination of 1311 oscillator and 1232 detector is available as the Type 1240 Bridge Oscillator-Detector.
DC Polarization: Capacitors can be biased to 600 V from external dc power supply for series capacitance measurements.
Power Required: 4 size-D cells, supplied.
Accessories Required: None. Earphones can be used for high precision at extremes of bridge ranges.

Accessories Available: Type 1650-P1 Test Jig.
Mounting: Flip-Tilt Cabinet.
Dimensions (width $\times$ height $\times$ depth): Portable, $13 \times 63 / 4 \times 121 / 4 \mathrm{in}$. $(330 \times 175 \times 315 \mathrm{~mm})$; rack, $19 \times 12 \frac{1 / 4}{} \times 4 \frac{1 / 8}{} \mathrm{in}$. $(485 \times 315 \times$ 105 mm ).
Net Weight (est): Portable, $17 \mathrm{lb}(8 \mathrm{~kg})$; rack, $18 \mathrm{lb}(8.5 \mathrm{~kg})$.
Shipping Weight (est): Portable, $21 \mathrm{lb}(10 \mathrm{~kg}$ ); rack, 30 lb ( 13.5 kg ).

| Catalog <br> Number | Description |  |
| :---: | :--- | :--- |
| $1650-9702$ | 1650-B Impedance Bridge <br> Portable Model <br> $1650-9703$ | Rack Model |

